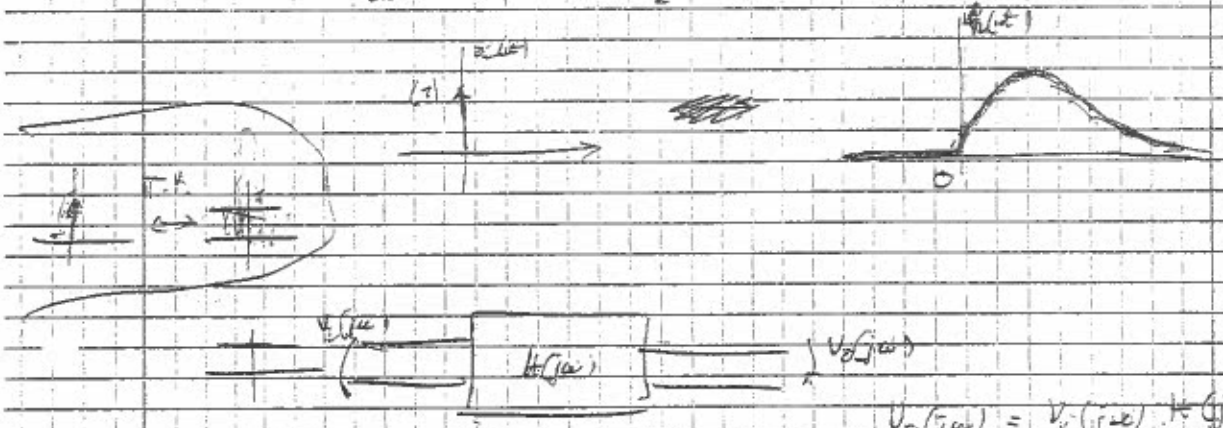
$$H(j\omega) = \frac{\frac{1}{2}}{j\omega+1} + \frac{1}{j\omega+2} - \frac{\frac{3}{2}}{j\omega+3}$$

$$h(t) = \left[\frac{1}{2}e^{-t} + e^{-2t} - \frac{3}{2}e^{-3t}\right]u(t)$$

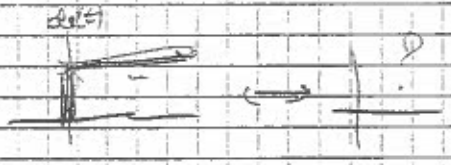


$$V_o(j\omega) = V_i(j\omega) \cdot H(j\omega)$$

$$\text{se } V_i(j\omega) = 1$$

$$|V_o(j\omega)| = |H(j\omega)| \longrightarrow h(t)$$

b)



$$d_T(t) = d_1(t) - d_2(t) =$$

$$= d_1(t) - d_1(t-T)$$

$$V_o(j\omega) = \frac{2j\omega - 3}{(j\omega+1)(j\omega+2)(j\omega+3)} \cdot \underbrace{\left[\frac{1}{j\omega} - \pi\delta(\omega)\right]}_{\text{transf. de degrau}} =$$

$$= \frac{2j\omega - 3}{(j\omega+1)(j\omega+2)(j\omega+3)(j\omega)} + \pi\frac{0+3}{(0+1)(0+2)(0+3)}\delta(\omega) = \frac{|2|\omega+3}{---} + \left(\frac{3}{6}\right)\pi\delta(\omega)$$

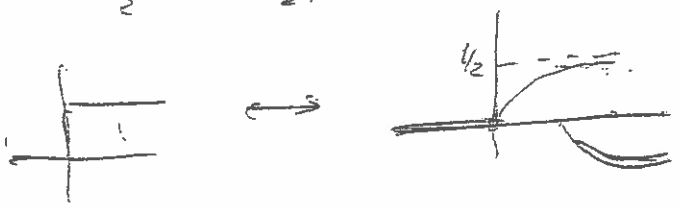
$$u(t) = \left[\frac{1}{2}e^{-t} + e^{-2t} - \frac{1}{2}e^{-3t}\right]u(t)$$

$$D_1(t) = \left(-\frac{1}{2}e^{-t} - \frac{1}{2}e^{-2t} + \frac{1}{2}e^{-3t} + C_1\right)u(t) \qquad D_1(0) = 0$$

$$-\frac{1}{2} + C_1 = 0$$

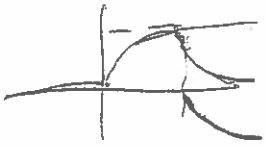
$$C_1 = 1/2$$

$$D_1(t) = \left(-\frac{1}{2}e^{-t} - \frac{1}{2}e^{-2t} + \frac{1}{2}e^{-3t} + \frac{1}{2}\right).u(t)$$



Como o sistema é Invariante no tempo

$$D_2(t) = \left(\frac{1}{2}e^{-(t-T)} + \frac{1}{2}e^{-2(t-T)} - \frac{1}{2}e^{-3(t-T)} - \frac{1}{2}\right).u(t-T)$$



$$D(t) = D_1(t) + D_2(t)$$

e) $x(t) = 5.e^{-4t}.u(t)$

$$Y(j\omega) = \frac{2j\omega + 3}{(j\omega+1)(j\omega+2)(j\omega+3)} \cdot 5 \cdot \frac{1}{j\omega+4} = \frac{A}{j\omega+1} + \frac{B}{j\omega+2} + \frac{C}{j\omega+3} + \frac{D}{j\omega+4}$$

$$A = \left.\frac{5(2n+3)}{(n+2)(n+3)(n+4)}\right|_{n=-1}$$

$$y(t) = \left(Ae^{-t} + Be^{-2t} + Ce^{-3t} + De^{-4t}\right).u(t)$$

f) $$Y(j\omega) = \frac{2j\omega+3}{(j\omega+1)^3(j\omega+2)(j\omega+3)} =$$

$$= \frac{A}{(j\omega+1)^3} + \frac{B}{(j\omega+1)^2} + \frac{C}{j\omega+1} + \frac{D}{j\omega+2} + \frac{E}{j\omega+3}$$

$$e^{-t}.u(t) \Leftrightarrow \frac{1}{j\omega+1}$$

$$t.e^{-t}.u(t) \Leftrightarrow \frac{1}{(j\omega+1)^2}$$

$$y(t) = \left(A\frac{t^2}{2}e^{-t} + Bte^{-t} + Ce^{-t} + De^{-2t} + Ee^{-3t}\right)\cdot u(t)$$

Novo exercício

e) $x(t) = 3\cos(3t) - 5\,\text{sen}(4t)$

$\omega = 3$ $\qquad$ $\omega = 4$

$\cos(3t) \longleftrightarrow \pi\left[\delta(\omega-3) + \delta(\omega+3)\right]$

$\text{sen}\,4t \longleftrightarrow \frac{\pi}{j}\left[-\delta(\omega-4) + \delta(\omega+4)\right]$

$$y(t) = 3\,|H(j3)|\cos(3t + \phi(3j)) - 5\,|H(j4)|\,\text{sen}(4t + \phi(4j))$$

$$H(j\omega) = \frac{2j\omega + 3}{(j\omega)^3 + 5j\omega^2 + 18j\omega + 6}$$

$j^0 = 1$
$j^1 = j$
$j^2 = -1$
$j^3 = -j$
$j^4 = 1$
$j^5 = j$

$$H(j3) = \frac{3+6j}{-27j - 54 + 33j + 6} = \frac{3+6j}{(-48) + 3j}$$

$$|H(j3)| = \frac{\sqrt{3^2+6^2}}{\sqrt{(-48)^2 + 3^2}}$$

$\phi(j3) = \ ?$

$|z| = \sqrt{z\bar{z}}$

$(3+4j)(3-4j)$

$$\frac{3+6j\,(-48-3j)}{(-48+3j)(-48-3j)} = \frac{-144 - 9j - 288j + 18}{(-48)^2 + 3^2} = \frac{-126 - 279j}{}$$

$$= \frac{A + jB}{}$$

$$\phi(3) = \text{arctg}\,\frac{-279}{-126}$$

$H(j4)$

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CURSO

DATA

$$y[n] + \frac{2}{15} y[n-1] - \frac{1}{15} y[n-2] = x[n] - x[n-1]$$

$$y[n] = Y(e^{j\omega})$$
$$x[n] = X(e^{j\omega})$$
$$x[n-1] = e^{-j\omega} X(e^{j\omega})$$
$$y[n-1] = e^{-j\omega} Y(e^{j\omega})$$
$$y[n-2] = e^{-2j\omega} Y(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 + \frac{2}{15} e^{-j\omega} - \frac{1}{15} e^{-2j\omega}\right] =$$

$$= X(e^{j\omega})\left[1 - e^{-j\omega}\right]$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 + \frac{2}{15} e^{-j\omega} - \frac{1}{15} e^{-2j\omega}} =$$

$$\frac{1}{3} \quad \frac{1}{5}$$

$$= \frac{1 - e^{-j\omega}}{\left(1 + \frac{1}{3} e^{-j\omega}\right)\left(1 - \frac{1}{5} e^{-j\omega}\right)} =$$

$$\frac{1}{3} \cdot \frac{1}{5} = \frac{2}{15}$$
$$(5) \quad (2)$$

$$= \frac{A}{1 + \frac{1}{3} e^{-j\omega}} + \frac{B}{1 - \frac{1}{5} e^{-j\omega}} =$$

$$\left(1 - \frac{1}{5} e^{-j\omega}\right) \quad \left(1 + \frac{1}{3} e^{-j\omega}\right)$$

$$r^2 + \frac{2}{15} r - \frac{1}{15} = 0$$

$$A - \frac{A}{5} e^{-j\omega} + B + \frac{B}{3} e^{-j\omega} = 1 - e^{-j\omega}$$

$$r = \frac{-\frac{2}{15} \pm \sqrt{\frac{4}{225} + \frac{4}{15}}}{2} =$$

$$\begin{cases} A + B = 1 \\ -\frac{A}{5} + \frac{B}{3} = -1 \end{cases}$$

$$= \frac{-\frac{2}{15} \pm \sqrt{\frac{64}{225}}}{2} = \frac{-\frac{2}{15} \pm \frac{8}{15}}{2}$$

$$\begin{cases} A = \\ B = \end{cases}$$

$$= \frac{\frac{6}{15}}{2} = \frac{6}{30} = \frac{1}{5}$$

$$\frac{10}{30} = -\frac{1}{3}$$

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PROVA DA DISCIPLINA

CURSO

DATA

$$\frac{A}{1-\frac{1}{3}e^{-j\omega}} \longleftrightarrow \left[\left(\frac{1}{3}\right)^{n} \cdot A\right] \cdot u[n]$$

$$\frac{B}{1+\frac{1}{5}e^{-j\omega}} \longleftrightarrow \left[B \cdot \left(-\frac{1}{5}\right)^{n}\right] \cdot u[n]$$

DEPARTAMENTO DE FÍSICA E ASTRONOMIA

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