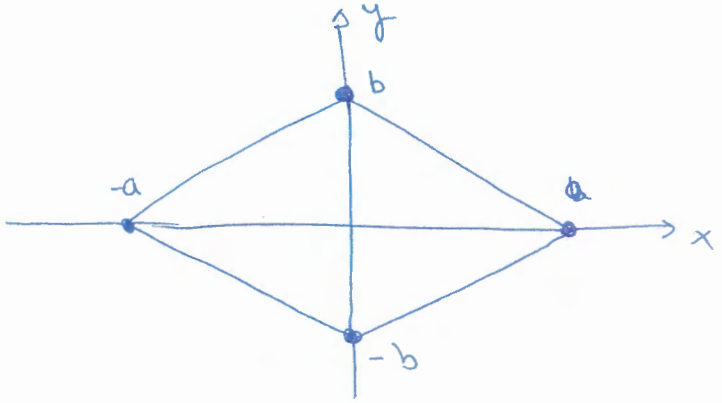
1

a)



$a, b > 0$

$\vec{E}(0,0) = \vec{0}$ (configuração simétrica

tirando a $(\pm a, 0)$: $\vec{E}(0,0) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{a^2}(\pm\hat{x})$

tirando a $(0, \pm b)$: $\vec{E}(0,0) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{b^2}(\pm\hat{y})$

b) $$\phi(0,0) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{a} + \frac{1}{b} + \frac{1}{b}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{b}\right)$$

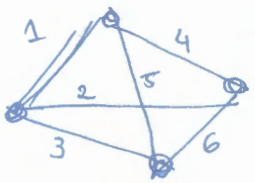
c) $$U = \frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ i\neq j}}^{N}\frac{Q_i Q_j}{|\vec{r}_i - \vec{r}_j|}$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j>i}}^{N}\frac{Q_i Q_j}{|\vec{r}_i - \vec{r}_j|}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{a^2+b^2}} + \frac{1}{2a} + \frac{1}{\sqrt{a^2+b^2}} + \frac{1}{\sqrt{a^2+b^2}} + \frac{1}{2b} + \frac{1}{\sqrt{a^2+b^2}}\right)$$

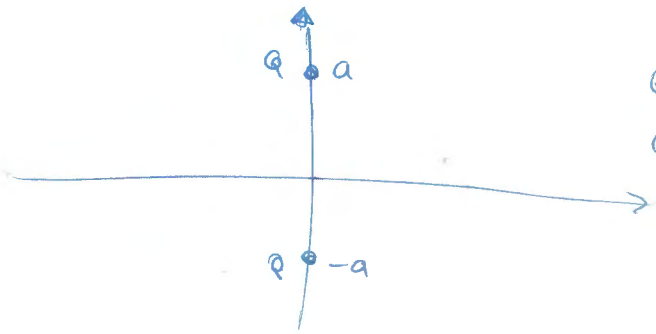
6 termos ($^4C_2 = 6$) ✓

correspondentes às ligações não repetidas entre as cargas



(2)

a)



$Q: (0,a)$
$Q: (0,-a)$

(a ≠ 0)

$$\vec{E}(x,y) = \frac{Q}{4\pi\varepsilon_0}\left\{ \frac{x\hat{x} + (y-a)\hat{y}}{[x^2 + (y-a)^2]^{3/2}} + \frac{x\hat{x} + (y+a)\hat{y}}{[x^2 + (y+a)^2]^{3/2}} \right\}$$

$$\vec{E}(x,0) = \frac{Q}{4\pi\varepsilon_0} \frac{2x\hat{x}}{[x^2+a^2]^{3/2}}$$

$$f(a) = \frac{2x}{[x+a^2]^{3/2}}$$

$$f(0) = \frac{2x}{x^3} = \frac{2}{x^2}$$

$a \ll x$, $a = 0$

$$f(a) \approx f(0) + f'(0)(a-0) + \dots$$

$$f(a) \approx f(0) = \frac{2}{x^2}$$

$$\boxed{\frac{2x}{[x^2+a^2]^{3/2}} \approx \frac{2}{x^2} \quad (a \ll x)}$$

$$\vec{E}(x,0) \approx \frac{2Q}{4\pi\varepsilon_0 x^2}\hat{x}$$

b) (q,m) em $(0,0)$ com $\vec{v} = v_0\hat{x}$ $v_0 > 0$

Usando que $\vec{E}(x,0) = \frac{Q}{4\pi\varepsilon_0} \frac{2x}{(x^2+a^2)^{3/2}}\hat{x}$

agora aproximando para $x \ll a$ fazemos:

$$f(x) = \frac{2x}{(x^2+a^2)^{3/2}}$$

$$f'(x) = \frac{2[x^2+a^2]^{3/2} - 3x^2(x^2+a^2)^{1/2}}{(x^2+a^2)^3}$$

$$f(x) = f(0) + f'(0)(x-0) + \dots$$

$$f(0) = 0$$

$$f'(0) = \frac{2}{a^3}$$

$$f(x) \approx 0 + \frac{2}{a^3}x + \dots$$

$$\boxed{\frac{2x}{(x^2+a^2)^{3/2}} \approx \frac{2x}{a^3} \quad (x \ll a)}$$

$$\vec{E}(x,0) \approx \frac{2Q}{4\pi\varepsilon_0 a^3} x\,\hat{x} \qquad (x \ll a)$$

$$\vec{F} = m\frac{d^2\vec{x}}{dt^2} = m\ddot{\vec{x}} = q\vec{E}(x,0)$$

Considerando só o eixo dos xx

temos

$$m\ddot{x} = qE_x(x,0) = \frac{qQ}{2\pi\varepsilon_0 a^3} x$$

Como $q<0$ e $Q>0$ $\quad qQ<0$

e $\quad -|q||Q| = qQ$

e assim temos

$$m\ddot{x} = -\frac{|q||Q|}{2\pi\varepsilon_0 a^3} x \iff \ddot{x} + \frac{|q||Q|}{2\pi\varepsilon_0 a^3 m} x = 0$$

$$\omega_0^2 = \frac{|q||Q|}{2\pi\varepsilon_0 a^3 m} \qquad x(t) = x_0 \sin(\omega_0 t + \varphi)$$

$x(0)=0 \Rightarrow \varphi = 0 \Rightarrow x(t) = x_0 \sin(\omega_0 t)$

$x'(0) = v_0 \qquad x'(t) = x_0\omega_0 \cos(\omega_0 t)$

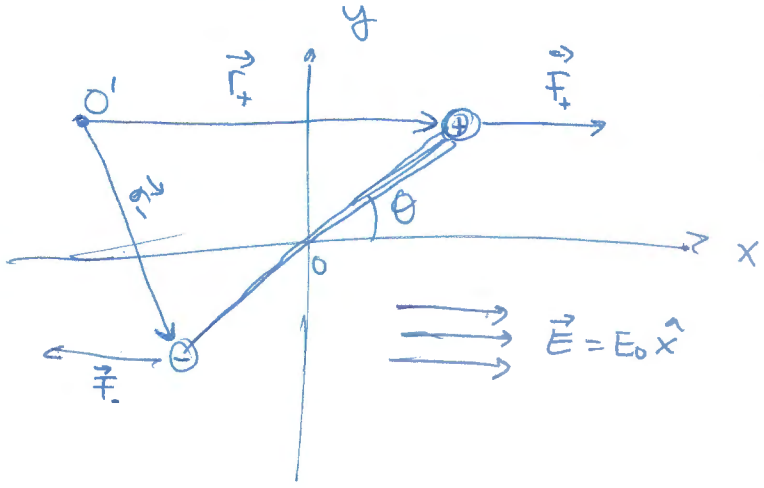
$$\Rightarrow v_0 = x_0\omega_0 \Rightarrow \boxed{x_0 = \frac{v_0}{\omega_0}} \qquad x''(t) = -x_0\omega_0^2 \sin(\omega_0 t) = -v_0\omega_0 \sin(\omega_0 t)$$

c) $E = \frac{1}{2}m\dot{x}^2 + \underbrace{\frac{1}{2}kx^2}_{V(x)} \overset{(\cdots)}{=} \frac{1}{2}m\omega_0^2 x_0^2 = \frac{1}{2}m\omega_0^2\frac{v_0^2}{\omega_0^2} = \frac{1}{2}mv_0^2$

(energia dada inicial)

③

a)



θ ângulo que o braço "positivo" faz com $\hat{x}$

$$\vec{N}_O = \vec{N}_{O'} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- \qquad (\vec{N} \text{ torque})$$

como $\vec{F}_+ = -\vec{F}_-$

$$\vec{N}_O = \vec{N}_{O'} = (\vec{r}_+ - \vec{r}_-) \times \vec{F}_+ = (\vec{r}_+ - \vec{r}_-) \times |q| \vec{E}$$

$$= |q| (\vec{r}_+ - \vec{r}_-) \times \vec{E}$$

braço $\vec{d}$

$$q\vec{d} = \vec{p}$$

$$= \vec{p} \times \vec{E}$$

$$\vec{N} = pE_0 \sin\theta (\pm\hat{z}) \quad \rightarrow \text{depende da orientação das cargas}$$

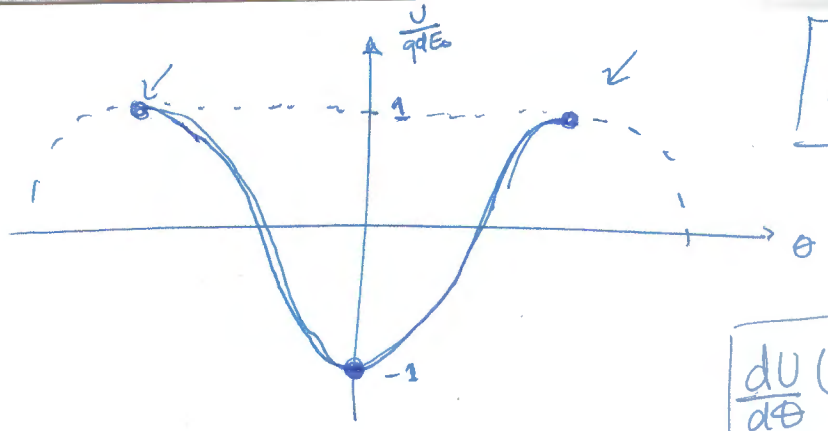
$$= qd \sin\theta (\pm\hat{z}) \qquad \pm\hat{z} \text{ corresponde a ser } \underline{\theta} \text{ ou } \underline{\theta+\pi}$$

o ângulo

$$dV = q[V(\vec{r}+d\vec{r}) - V(\vec{r})] = q[V(\vec{r}) + \nabla V(\vec{r}) \cdot d\vec{r} - V(\vec{r})]$$

$$dV = -q\vec{E} \cdot d\vec{r} \qquad \vec{E} = -\nabla V(\vec{r})$$

$$V = -\vec{p} \cdot \vec{E} = -qdE_0 \cos\theta \qquad \theta = [-\pi, \pi]$$



$$\frac{U}{qdE} = -\cos\theta$$

$$\frac{dU}{d\theta}(\theta) = 0$$

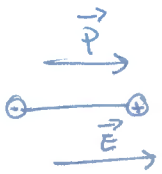
↳ equilíbrios

em $\theta = \pm\pi$ (mesma configuração na realidade)

$U = qdE_0$

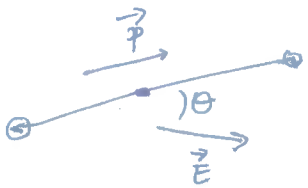
a energia é máxima e são pontos de equilíbrio instável

em $\theta = 0$ o equilíbrio é estável (energia mínima)



$U = -qdE_0$

b)



$\vec{E} = E_0\hat{x}$

$\vec{p} = qd\hat{p}$

$\theta(\theta) = \theta_0$

$\hat{x}\cdot\hat{p} = \cos\theta$

$$\vec{N} = \vec{p}\times\vec{E} = qdE_0\sin\theta\,\hat{z}$$

$$\Rightarrow -I_{cm}\frac{d^2\theta}{dt^2} = qdE_0\sin\theta$$

para θ pequeno $\sin\theta \approx \theta$

$$\Rightarrow \ddot{\theta} + \frac{qdE_0}{I_{cm}}\theta = 0$$

OHS

$$I_{cm} = \sum_i d_i^2 m_i = \left(\frac{d}{2}\right)^2 m + \left(\frac{d}{2}\right)^2 m = \frac{1}{2}d^2 m$$

$$\omega_0^2 = \frac{qdE_0}{I_{cm}} = \frac{2qdE_0}{d^2 m} \Rightarrow 2\pi f = \sqrt{\frac{2qE_0}{dm}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{2qE_0}{dm}}}$$

$$\theta(t) = \theta_0\cos(\omega_0 t)$$

c) $\dot{\theta}(t) = -\theta_0 \omega_0 \sin(\omega_0 t)$

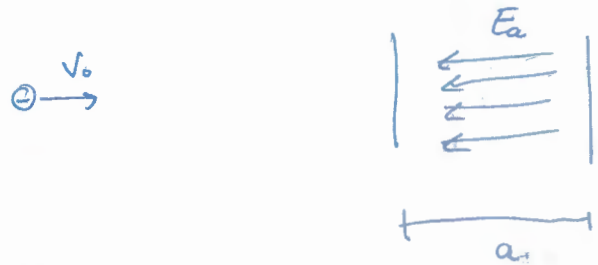
$$K = \frac{1}{2} I \dot{\theta}^2(t) = \frac{1}{4} d^2 m \theta_0^2 \omega_0^2 \sin^2(\omega_0 t)$$

quando passa na posição de eq. K é máximo

$$K = \frac{1}{4} d^2 m \theta_0^2 \omega_0^2 = \frac{1}{2} q d E_0 \theta_0^2$$

(4)

a)



caso 1

$V_{AB} = E_a \cdot a$

$\vec{V}_0 = V_0 \hat{x}$

$\vec{E}_a = -E_a \hat{x}$ $\quad \vec{F}_a = -e\vec{E} = eE_a\hat{x} \Rightarrow \vec{a} = \frac{eE_a}{m_e}\hat{x}$

$$x(t) = V_0 t + \frac{eE_a}{2m_e} t^2$$

$$V(t) = V_0 + \frac{eE_a}{m_e} t$$

$x(t_a) = a \Rightarrow \frac{eE_a}{2m_e} t_a^2 + V_0 t_a - a = 0$

$$t_a^2 + \frac{2m_e V_0}{eE_a} t - \frac{2m_e a}{eE_a} = 0$$

$$t_a = -\frac{2m_e V_0}{2eE_a} \pm \frac{1}{2}\sqrt{\left(\frac{2m_e V_0}{eE_a}\right)^2 + 4\frac{2m_e a}{eE_a}}$$

$$t_a = -\frac{m_e V_0}{eE_a} + \sqrt{\left(\frac{m_e}{eE_a}\right)^2 \left(2a\frac{eE_a}{m_e} + V_0^2\right)}$$

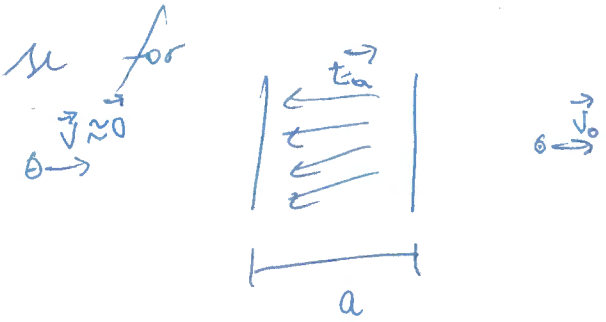
$$t_a = \frac{m_e}{eE_a}\left[\underbrace{\sqrt{2a\frac{eE_a}{m_e} + V_0^2}}_{V_{final}} - V_0\right] = \frac{\Delta V}{|\vec{a}|}$$

$$v_f = \sqrt{2a\frac{eE_a}{m_e} + v_0^2}$$

$$K = \frac{1}{2}m_e v_f^2 = \frac{1}{2}m_e v_0^2 + eE_a a$$

$$= \frac{1}{2}m_e v_0^2 + eV_{ab}$$

Se for



Caso 2

como na imagem

então $K = (eE_a)a = \frac{1}{2}m_e v_0^2$

$$\frac{1}{2}m_e v_0^2 = eE_a a \Rightarrow v_0 = \sqrt{\frac{2eaE_a}{m_e}}$$

$$t_a = \frac{\Delta v}{|\vec{a}|} = \frac{v_0}{(eE_a/m_e)} = \sqrt{\frac{2eE_a a}{m_e}\left(\frac{m_e}{eE_a}\right)^2} = \sqrt{2a\frac{m_e}{eE_a}}$$

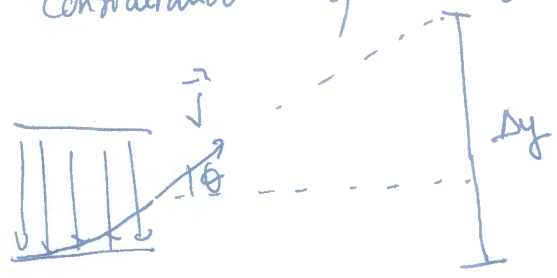
b) $x = v_a t$

$$y = \frac{eE_0}{2m_e}t^2 \qquad v_y = \frac{eE_0}{m_e}t \Rightarrow \vec{v_d} = v_a\hat{x} + \frac{eE_0 d}{m_e v_a}\hat{y}$$

$$\Rightarrow y = \frac{eE_0 x^2}{2m_e v_a^2} \Rightarrow y_d = \frac{eE_0 d^2}{2m_e v_a^2}$$

$$t_d = \frac{d}{v_a}$$

c) considerando que $v_a = \sqrt{2a\frac{E_a e}{m_e} + v_0^2}$



$$\tan\theta = \frac{(\vec{v_d})_y}{(\vec{v_d})_x} = \frac{eE_0 d}{m_e v_a^2}$$

$$\Delta y = y_d + L\tan\theta$$

$$\Delta y = \frac{eE_0 d^2}{2 m_e V_a^2} + (2L)\frac{eE_0 d}{2 m_e V_a^2}$$

$$\Delta y = \frac{eE_0 d}{2 m_e V_a^2}(d+2L) = \frac{eE_0 d}{2 m_e V_0^2 + 4aE_a e}(d+2L)$$

$$\Delta y = \frac{eE_0 d}{\frac{1}{2}m_e V_0^2 + eE_a d}\left(\frac{d}{4}+\frac{L}{2}\right)$$

~~$K = \frac{1}{2}m_e V_0^2 + eE_0 d + eE_a a$~~

$$K = \frac{1}{2}m_e V_d^2 = \frac{1}{2}m_e\left(V_{dx}^2 + V_{dy}^2\right)$$

$$= \frac{1}{2}m_e\left[\frac{2aE_a e}{m_e} + V_0^2 + \left(\frac{eE_0 d}{m_e V_a^2}\right)^2\right]$$

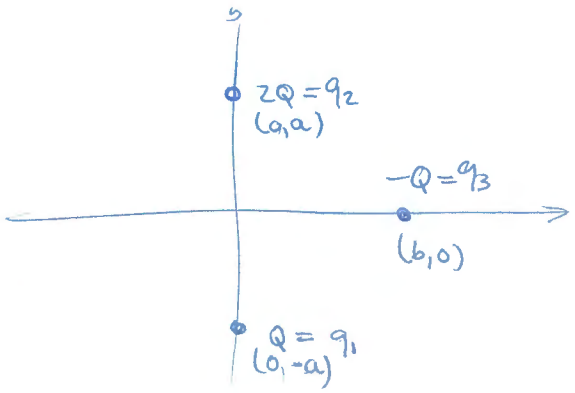
$$= \frac{1}{2}m_e V_0^2 + aE_a e + \frac{1}{2}m_e\left(\frac{eE_0 d}{m_e}\right)^2 \frac{1}{2aE_a\frac{e}{m_e} + V_0^2}$$

$$= \frac{1}{2}m_e V_0^2 + aE_a e + \frac{1}{2}\frac{e^2 E_0^2 d^2}{2aE_a e + m_e V_0^2}$$

inicial — zona de aceleração — zona de desvio

(5)

a)



$a, b > 0$

$$\vec{E}(x,y) = \frac{1}{4\pi\varepsilon_0}\left\{ q_1 \frac{x\hat{x} + (y+a)\hat{y}}{[x^2 + (y+a)^2]^{3/2}} + q_2 \frac{x\hat{x} + (y-a)\hat{y}}{[x^2 + (y-a)^2]^{3/2}} + q_3 \frac{(x-b)\hat{x} + y\hat{y}}{[(x-b)^2 + y^2]^{3/2}} \right\}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left\{ \frac{x\hat{x} + (y+a)\hat{y}}{[x^2 + (y+a)^2]^{3/2}} + 2\frac{x\hat{x} + (y-a)\hat{y}}{[x^2 + (y-a)^2]^{3/2}} - \frac{(x-b)\hat{x} + y\hat{y}}{[(x-b)^2 + y^2]^{3/2}} \right\}$$

$$V(x,y) = \phi(x,y) = \frac{Q}{4\pi\varepsilon_0}\left\{ \frac{1}{[x^2 + (y+a)^2]^{1/2}} + \frac{2}{[x^2 + (y-a)^2]^{1/2}} - \frac{1}{[(x-b)^2 + y^2]^{1/2}} \right\}$$

$$\vec{E}(0,y) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(y+a)^2}\hat{y} + \frac{2}{(y-a)^2}\hat{y} + \frac{b\hat{x} - y\hat{y}}{(b^2 + y^2)^{3/2}} \right]$$

$$\phi(0,y) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{|y+a|} + \frac{2}{|y-a|} - \frac{1}{(b^2 + y^2)^{1/2}} \right]$$

$a, b \ll |y| \rightarrow$ aproximar para $a, b \to 0$

$$f(a) = \frac{1}{(y+a)^2} + \frac{2}{(y-a)^2} \approx f(0) = \frac{3}{y^2}$$

$$g(b) = \frac{b}{(b^2 + y^2)^{3/2}} \approx g(0)^{0} + g'(0)\,b = \frac{b}{|y|^3} \text{ (podendo ser aproximada a 0)}$$

$$h(b) = \frac{y}{(b^2 + y^2)^{3/2}} \approx h(0) = \frac{y}{|y|^3} = \frac{y}{y^2|y|} = \frac{1}{y|y|}$$

$$\hookrightarrow \vec{E}(0,y) \approx \frac{Q}{4\pi\varepsilon_0}\left[\left(\frac{3}{y^2} - \frac{1}{|y|^3}\right)\hat{y} + \frac{b}{|y|^3}\hat{x} \right] \approx \frac{Q}{4\pi\varepsilon_0}\left(\frac{3}{y^2} - \frac{1}{y|y|} \right)\hat{y}$$

para aproximar $\phi(0,y)$

$$p(a) = \frac{1}{|y+a|} + \frac{2}{|y-a|} \approx p(0) = \frac{3}{|y|}$$

$$q(b) = \frac{1}{(b^2+y^2)^{1/2}} \approx q(0) = \frac{1}{|y|}$$

$$\phi(0,y) \approx \frac{Q}{4\pi\varepsilon_0}\left(\frac{3}{|y|} - \frac{1}{|y|}\right) = \frac{2Q}{4\pi\varepsilon_0|y|}$$

c) $$U_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{a} + \frac{q_2q_3}{b} + \frac{q_3q_1}{\sqrt{a^2+b^2}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q^2}{a} - \frac{2Q^2}{b} - \frac{Q^2}{\sqrt{a^2+b^2}}\right)$$

$$= \frac{Q^2}{2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b} - \frac{1}{2\sqrt{a^2+b^2}}\right)$$

b) 1º método:

$$W = \Delta U = U_f - U_i$$

$$U_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{2a} + \frac{q_2q_3}{\sqrt{a^2+b^2}} + \frac{q_1q_3}{\sqrt{a^2+b^2}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q^2}{2a} - \frac{2Q^2}{\sqrt{a^2+b^2}} - \frac{Q^2}{\sqrt{a^2+b^2}}\right)$$

$$= \frac{Q^2}{2\pi\varepsilon_0}\left(\frac{1}{2a} - \frac{3}{2\sqrt{a^2+b^2}}\right)$$

$$W = U_f - U_i = \frac{Q^2}{2\pi\varepsilon_0}\left(\frac{1}{2a} - \frac{1}{b} + \frac{1}{\sqrt{a^2+b^2}}\right)$$

2º método:

Potencial em relação a q_1 e q_3

$$\phi_{13}(0,y) = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{|y+a|} - \frac{1}{\sqrt{b^2+y^2}}\right) \quad \text{(tirar o termo da } q_2\text{)}$$

$$W_{a\to b} = qV_{ab} = 2Q\left[\phi_{13}(0,0) - \phi_{13}(0,a)\right]$$

$$= \frac{2Q^2}{4\pi\varepsilon_0}\left(\frac{1}{2a} - \frac{1}{b} + \frac{1}{\sqrt{a^2+b^2}}\right)$$

(6)

Assumindo que:

$\vec{p}_0 = \vec{p}_\alpha + \vec{p}$

$E_{K0} = \frac{p_0^2}{2m_\alpha}$

$m_\alpha = 2m_p + 2m_n \simeq 4m_p$

$m = 79m_p + 118m_n \simeq 197m_p$

$$E_{K0} = U(d_{min}) = U(d) + \frac{p_\alpha^2}{2m_\alpha} + \frac{p^2}{2m}$$

a) $E_{K0} = U(d_{min}) = \frac{(2e)(79e)}{4\pi\varepsilon_0 d_{min}} \Rightarrow d_{min} = \frac{79e^2}{2\pi\varepsilon_0 E_{K0}}$

$p_\alpha p_0 = \frac{1}{3}p$

b) $|\vec{p}_\alpha| = \frac{1}{3}|\vec{p}_0| \Rightarrow \frac{p_\alpha^2}{2m_\alpha} = \left(\frac{p_0^2}{2m_\alpha}\right)\frac{1}{9} = \frac{1}{9}E_{K0}$

$$\frac{p^2}{2m} = \frac{(p_0-p_\alpha)^2}{2m} = \left(\frac{m_\alpha}{m}\right)\left[\frac{p_0^2}{2m_\alpha} + \frac{p_\alpha^2}{2m_\alpha} - \frac{2p_0p_\alpha}{2m_\alpha}\right]$$

$$= \left(\frac{m_\alpha}{m}\right)\left[E_{K0} + \frac{1}{9}E_{K0} - \frac{2}{3}E_{K0}\right]$$

$$= \frac{4}{9}\left(\frac{m_\alpha}{m}\right)E_{K0}$$

$p_0 = \sqrt{2m_\alpha E_{K0}}$

$p_\alpha = \sqrt{2m_\alpha\left(\frac{1}{9}E_{K0}\right)}$

$p_0p_\alpha = \frac{2}{3}m_\alpha E_{K0}$

$$E_{K_0} = U(d) + \frac{p_x^2}{2m_\alpha} + \frac{p^2}{2m}$$

$$E_{K_0} = U(d) + \frac{1}{9}E_{K_0} + \frac{4}{9}\left(\frac{m_\alpha}{m}\right)E_{K_0}$$

$$U(d) = E_{K_0}\left[1 - \frac{1}{9} - \frac{4}{9}\left(\frac{m_\alpha}{m}\right)\right] = E_{K_0}\left(\frac{8}{9} - \frac{4m_\alpha}{9m}\right)$$

$$\Rightarrow d = \frac{79e^2}{2\pi\varepsilon_0 E_{K_0}}\left(\frac{9m}{8m - 4m_\alpha}\right)$$

Considerando que $\boxed{m \gg m_\alpha}$ e que a partícula de ouro não se mexe em relação à α, tomamos

$$\lim_{m \to \infty}\left(\frac{9m}{8m - 4m_\alpha}\right) = \frac{9}{8} \Rightarrow d \approx \frac{79e^2}{2\pi\varepsilon_0 E_{K_0}}\left(\frac{9}{8}\right)$$

c) $\vec{F} = m\vec{a} \Rightarrow \frac{79e^2}{2\pi\varepsilon_0 x^2} = m_\alpha a \Rightarrow a = \frac{79e^2}{2\pi\varepsilon_0 m_\alpha x^2}$

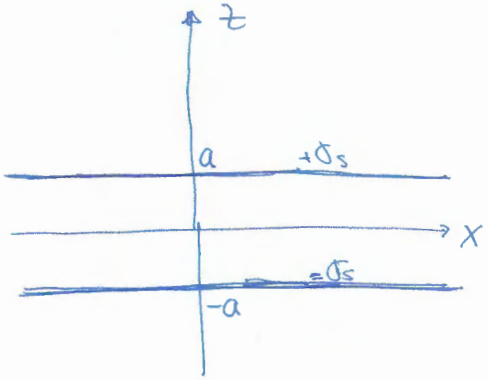
(Considerando que o núcleo de ouro não se move e x sendo a distância entre eles)

Como $a = \frac{d^2x}{dt} = \ddot{x}$ temos a eq. d.f:

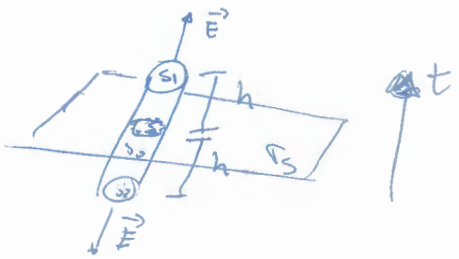
$$\ddot{x} = \left(\frac{79e^2}{2\pi\varepsilon_0 m_\alpha}\right)\frac{1}{x^2}$$

(7)

a)



$$\oint_{\partial V = S} \vec{E}\cdot d\vec{S} = \frac{Q(V)}{\varepsilon_0}$$



S_1: base superior S_2: base inferior S_3: superfície lateral

$$\oint_{S_1+S_2+S_3=\partial V} \vec{E}\cdot d\vec{S} = \frac{Q(V)}{\varepsilon_0} \Rightarrow \iint_{S_1}\vec{E}\cdot d\vec{S} + \iint_{S_2}\vec{E}\cdot d\vec{S} + \iint_{S_3}\vec{E}\cdot d\vec{S} = \frac{Q(V)}{\varepsilon_0}$$

Na S_3 o vector $d\vec{S}$ é perpendicular a $\vec{E}$ ↳ normal à superfície $\Rightarrow \vec{E}\cdot d\vec{S} = 0$

Na S_1 $\vec{E} = |\vec{E}|\hat{z}$ e $d\vec{S} = dS\,\hat{z}$

Na S_2 $\vec{E} = -|\vec{E}|\hat{z}$ e $d\vec{S} = -dS\,\hat{z}$ $\Big\rangle \vec{E}\cdot d\vec{S} = E\,dS$

Como as bases estão à mesma distância supõe-se que $|\vec{E}_{S_1}| = |\vec{E}_{S_2}|$, e o plano infinito significa que $\vec{E}$ não varia com (x,y)

$$\oint_{S_1+S_2+S_3}\vec{E}\cdot d\vec{S} = \iint_{S_1} E\,dS + \iint_{S_2} E\,dS = E\left(\iint_{S_1} dS + \iint_{S_2} dS\right) = E(2S)$$

$\text{Area}(S_1) = \text{Area}(S_2) = S$

$$\frac{Q(V)}{\varepsilon_0} = \frac{\sigma_s S}{\varepsilon_0} \Rightarrow E = \frac{\sigma_s}{2\varepsilon_0}$$

Variando h podemos concluir que

$$\vec{E}_{\pm a} = \begin{cases} |\vec{E}|\hat{z}, & z > a \\ -|\vec{E}|\hat{z}, & z < a \end{cases} = \begin{cases} \sigma_s/2\varepsilon_0\,\hat{z}, & z > a \\ -\sigma_s/2\varepsilon_0\,\hat{z}, & z < a \end{cases}$$

b) Pela mesma analogia e trocando σ_s por $(-\sigma_s)$

$$\vec{E}_{-a} = \begin{cases} -\frac{\sigma_s}{2\varepsilon_0}\hat{z}, & z > -a \\ \frac{\sigma_s}{2\varepsilon_0}\hat{z}, & z < -a \end{cases}$$

$$V = \left| -\int_C \vec{E} \cdot d\vec{l} \right| = \frac{\sigma_s}{\varepsilon_0} 2a$$

$$\vec{E} = \vec{E}_{+a} + \vec{E}_{-a} = \begin{cases} 0, & z > a \\ -\frac{\sigma_s}{\varepsilon_0}\hat{z}, & |z| < a \\ 0, & z < -a \end{cases}$$

$$\boxed{V = \left(\frac{\sigma_s}{\varepsilon_0} 2a\right)}$$

c)

Imaginando que pode-se contabilizar a area infinita do plano S_∞

$$U_i = qV = \sigma_s S_\infty \frac{\sigma_s}{\varepsilon_0} 2a \Rightarrow \frac{U_i}{S_\infty} = \frac{\sigma_s^2}{\varepsilon_0} 2a$$

$$\frac{U_f}{S_\infty} = \frac{\sigma_s^2}{\varepsilon_0} 4a$$

trabalho por unidade de area

$$\hookrightarrow W_s = \frac{U_f - U_i}{S_\infty} = \underbrace{\frac{\sigma_s^2}{\varepsilon_0}}_{\text{pressão eletrostática}} (4a - 2a) = \frac{\sigma_s^2}{\varepsilon_0} 2a$$

⑧

$$\rho = \frac{-Ze}{\frac{4}{3}\pi R^3}$$

a) $Q(r<R) = Ze + \rho \frac{4}{3}\pi r^3$ $\qquad Q(r>R) = 0$

$$= Ze\left[1 - \left(\frac{r}{R}\right)^3\right]$$

b) $\varepsilon_0 \oiint_{\partial V} \vec{E}\cdot d\vec{S} = Q(V)$

$$\Rightarrow \varepsilon_0 4\pi r^2 E(r) = Ze\left[1-(r/R)^3\right]$$

$$\Rightarrow E(r) = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right) \qquad r<R$$

$$E(r) = 0 \quad , r>R$$

c)

$$E(r) = \begin{cases} \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right), & r<R \\ 0, & r>R \end{cases}$$

$$\phi(r) = -\int E(r)\,dr = \begin{cases} \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{r^2}{2R^3} + C_1\right), & r<R \\ C_2, & r \geq R \end{cases}$$

$r=R \Rightarrow \quad \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{R} + \frac{1}{2R} + C_1\right) = C_2$

considerando que $\phi(R)=0 \Leftarrow \phi(\infty)=0$

$C_2 = 0$

$C_1 = -\frac{3}{2R}$

(9)

a) $\varepsilon_0 \oint_{ar} \vec{E}(r)\cdot d\vec{S} = Q(V)$

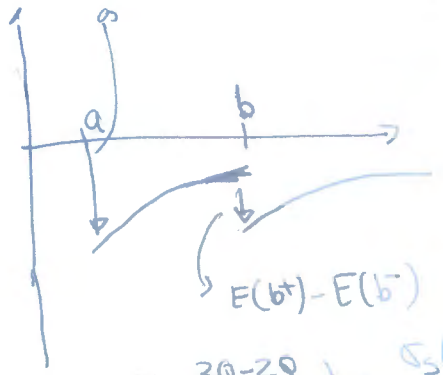
$\Downarrow$

$E(r) = 0 \quad , r < a$

$E(r) = \frac{2Q}{4\pi\varepsilon_0 r^2} \quad , a < r < b$

$E(r) = \frac{3Q}{4\pi\varepsilon_0 r^2} \quad , r > b$

$E(a) = \frac{2Q}{4\pi a^2}\frac{1}{\varepsilon_0} = \frac{\sigma_s(a)}{\varepsilon_0}$



$= \frac{3Q-2Q}{4\pi b^2}\frac{1}{\varepsilon_0} = \frac{\sigma_s(b)}{\varepsilon_0}$

b) $\phi(b) - \phi(a) = -\int_a^b E(r)\,dr = -\frac{2Q}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}dr = \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$

c) $U = \frac{1}{2}\varepsilon_0 \iiint_V E(r)^2\, dV = \frac{1}{2}\varepsilon_0 \iiint E(r)^2 r^2 \sin\theta\, dr\, d\theta\, d\varphi$

$= \frac{4\pi\varepsilon_0}{2}\int_0^{+\infty} E(r)^2 r^2 dr = \frac{4\pi\varepsilon_0}{2}\int_a^b \left(\frac{2Q}{4\pi\varepsilon_0}\right)^2 \frac{1}{r^4} r^2 dr +$

$+ \frac{4\pi\varepsilon_0}{2}\int_b^{+\infty}\left(\frac{3Q}{4\pi\varepsilon_0}\right)^2 \frac{1}{r^4} r^2 dr =$

$= \frac{1}{2}\left(\frac{4Q^2}{4\pi\varepsilon_0}\right)\int_a^b \frac{1}{r^2}dr + \frac{1}{2}\left(\frac{9Q^2}{4\pi\varepsilon_0}\right)\int_b^{+\infty}\frac{1}{r^2}dr$

$= \underbrace{\frac{Q^2}{2\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}_{Q[\phi(a)-\phi(b)]} + \frac{1}{2}\left(\frac{9Q^2}{4\pi\varepsilon_0}\right)\left(\frac{1}{b} - 0\right)$

$= \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{4}{a} - \frac{4}{b} + \frac{9}{b}\right) = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{4}{a} + \frac{5}{b}\right)$

(10) $\sigma_s = \frac{Q_s}{S} = \frac{Q_s}{\pi(b^2-a^2)}$

a) $V(0,0,z) = \int_0^{2\pi}\int_a^b \frac{\sigma_s\, ds}{4\pi\varepsilon_0} \frac{1}{[\rho_0^2+z^2]^{1/2}}$

$= \int_0^{2\pi}\int_a^b \frac{\sigma_s}{4\pi\varepsilon_0} \frac{\rho_0\, d\rho_0\, d\theta_0}{[\rho_0^2+z^2]^{1/2}}$

$= 2\pi \frac{\sigma_s}{4\pi\varepsilon_0} \int_a^b \frac{\rho_0}{[\rho_0^2+z^2]^{1/2}} d\rho_0$

$= \frac{\sigma_s}{2\varepsilon_0}\left[\sqrt{\rho_0^2+z^2}\right]_a^b = \frac{\sigma_s}{2\varepsilon_0}\left(\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right)$

b) $\vec{E}(0,0,z) = -\frac{dV(0,0,z)}{dz}\hat{z} = \frac{\sigma_s}{2\varepsilon_0}\left(\frac{z}{\sqrt{a^2+z^2}} - \frac{z}{\sqrt{b^2+z^2}}\right)\hat{z}$

c) $a, b \gg |z|$ $\quad \frac{1}{\sqrt{a^2+z^2}} \approx \frac{1}{|a|} = \frac{1}{a}$

$\frac{1}{\sqrt{b^2+z^2}} \approx \frac{1}{|b|} = \frac{1}{b}$

$\vec{E}(0,0,z) \approx \frac{\sigma_s z}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)\hat{z} = \frac{Q_s}{2\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)\frac{z}{(b-a)(b+a)}\hat{z}$

$\approx \frac{Q_s}{2\pi\varepsilon_0}\frac{z}{ab(a+b)}\hat{z}$

(11)

a) $\varepsilon_0 \oint_{\partial V} \vec{E}\cdot\vec{dS} = Q(V) \Rightarrow \varepsilon_0 (2\pi r) L\, E(r) = -\sigma_L L$

$$\Rightarrow \vec{E}(r) = -\frac{\sigma_L}{2\pi\varepsilon_0 r}\hat{r} \quad , \quad a<r<b$$

$$Q(r>b) \Rightarrow \vec{E}(r) = \vec{0}$$

b) $$V = -\int_a^b \frac{-\sigma_L}{2\pi\varepsilon_0 r}\, dr = \frac{\sigma_L}{2\pi\varepsilon_0}\left[\ln r\right]_a^b = \frac{\sigma_L}{2\pi\varepsilon_0}\ln(b/a)$$

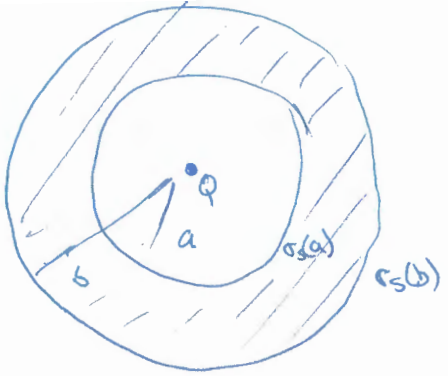
c) $$U_\infty = \frac{1}{2}\varepsilon_0 \underbrace{\int_{-\infty}^{+\infty} dz}_{L_\infty} \int_0^{2\pi} d\theta \int_0^{+\infty} E(r)^2 r\, dr$$

$$\Rightarrow U_L = \frac{U_\infty}{L_\infty} = \frac{1}{2}\varepsilon_0 (2\pi) \int_0^{+\infty} E(r)^2 r\, dr$$

$$U_L = \frac{2\pi\varepsilon_0}{2}\left(\frac{\sigma_L}{2\pi\varepsilon_0}\right)^2 \int_a^b \frac{1}{r^2} r\, dr$$

$$U_L = \frac{\sigma_L^2}{4\pi\varepsilon_0}\ln(b/a) = \frac{1}{2}\sigma_L V$$

12



a) Condutor $\vec{E}=\vec{0}$ no interior

dessa forma a carga desloca-se para anular o campo criado → a mesma carga simétrica "aparece" na superfície exterior

$$\vec{E}(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, \quad r<a$$

$$a<r<b \Rightarrow \varepsilon_0 \oiint \vec{E}(r)\cdot d\vec{s} = Q(V)$$

$$0 = Q(V) = Q + \sigma_s(a)\,4\pi a^2$$

$$\Rightarrow \sigma_s(a) = -\frac{Q}{4\pi a^2}$$

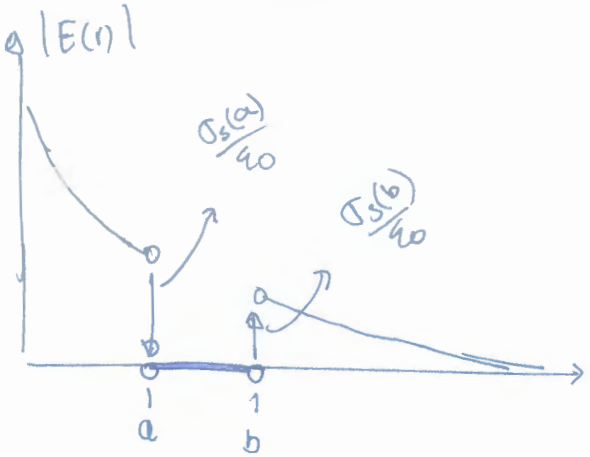
conservação da carga:

$$\sigma_s(a)\,4\pi a^2 + \sigma_s(b)\,4\pi b^2 = 0$$

$$-Q + \sigma_s(b)\,4\pi b^2 = 0$$

$$\Rightarrow \sigma_s(b) = \frac{Q}{4\pi b^2}$$

$$r>b \Rightarrow \vec{E}(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$



b)

$$E(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2}, & r < a \\ 0, & a < r < b \\ \frac{Q}{4\pi\varepsilon_0 r^2}, & r > b \end{cases}$$

$$\phi(r) = -\int E(r)\,dr = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r} + C_1, & r \leq a \\ C_2, & a < r \leq b \\ \frac{Q}{4\pi\varepsilon_0 r} + C_3, & r > b \end{cases}$$

$\boxed{\phi(r \to +\infty) = 0} \Rightarrow C_3 = 0$

$\phi(b^+) = \phi(b^-) \Rightarrow C_2 = \frac{Q}{4\pi\varepsilon_0 b}$

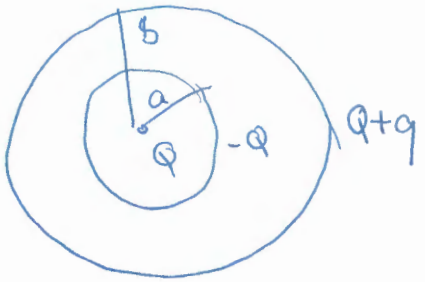
$\phi(a^+) = \phi(a^-) \Rightarrow \frac{Q}{4\pi\varepsilon_0 a} + C_1 = \frac{Q}{4\pi\varepsilon_0 b}$

$\Rightarrow C_1 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$

potencial função contínua

$$\phi(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{b} - \frac{1}{a}\right), & r \leq a \\ \frac{Q}{4\pi\varepsilon_0 b}, & a < r \leq b \\ \frac{Q}{4\pi\varepsilon_0 r}, & r > b \end{cases}$$

c)



$$E(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2} & , r < a \\ 0 & , a < r < b \\ \frac{Q+q}{4\pi\varepsilon_0 r^2} & , r > b \end{cases}$$

← sp zero dentro do condutor

Na superfície $r=a$ terá sempre $-Q$ independentemente do sinal de q

$$\phi(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right) + C_1 & , r \le a \\ C_2 & , a < r \le b \\ \frac{Q+q}{4\pi\varepsilon_0 r} + C_3 & , r \ge b \end{cases}$$

$\phi(r \to +\infty) = 0 \Rightarrow C_3 = 0$

$\phi(b^+) = \phi(b^-) \Rightarrow C_2 = \frac{Q+q}{4\pi\varepsilon_0 b}$

$\phi(a^+) = \phi(a^-) \Rightarrow C_1 + \frac{Q}{4\pi\varepsilon_0 a} = \frac{Q+q}{4\pi\varepsilon_0 b}$

$$C_1 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right) + \frac{q}{4\pi\varepsilon_0 b}$$

$$\phi(1) = \begin{cases} \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{b} - \frac{1}{a}\right) + \frac{q}{4\pi\varepsilon_0 b} & , r \le a \\ \frac{q+Q}{4\pi\varepsilon_0 b} & , a < r \le b \\ \frac{q+Q}{4\pi\varepsilon_0 r} & , r > b \end{cases}$$

(13)

a) $\vec{E}(r) = \begin{cases} 0 & , r<R \\ \frac{Q}{4\pi\varepsilon_0 r^2} & , r>R \end{cases}$

$$U = \frac{1}{2}\varepsilon_0 \iiint E^2 dV = \frac{4\pi\varepsilon_0}{2}\int_0^{+\infty} E^2(r)\, r^2 dr$$

$$= \frac{4\pi\varepsilon_0}{2}\left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \int_R^{+\infty} \frac{1}{r^4} r^2 dr = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{R} - 0\right) = \frac{Q^2}{8\pi\varepsilon_0 R}$$

b) $\rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$ $\qquad \varepsilon_0 \oiint \vec{E}(r)\cdot\vec{dS} = \rho \cdot \frac{4}{3}\pi r^3$

$$\Rightarrow 4\pi\varepsilon_0 r^2 E(r) = Q\left(\frac{r}{R}\right)^3$$

$$\Rightarrow E(r) = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3} \quad , r<R$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2} \quad , r>R$$

$$U = \frac{4\pi\varepsilon_0}{2}\left(\int_0^R \left(\frac{Q\,r}{4\pi\varepsilon_0 R^3}\right)^2 r^2 dr + \int_R^{+\infty}\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right) r^2 dr\right)$$

$$= \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\int_0^R \frac{r^4}{R^6} dr + \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$= \frac{Q^2}{8\pi\varepsilon_0 R}\left(\left[\frac{r^5}{5R^5}\right]_0^R + 1\right) = \frac{Q^2}{8\pi\varepsilon_0 R}\cdot\left(\frac{6}{5}\right)$$

c)

[a] $U = \frac{1}{2}QV \Rightarrow V = \frac{Q}{4\pi\varepsilon_0 R}$

[b] $V = \frac{Q}{4\pi\varepsilon_0 R}\left(\frac{6}{5}\right)$

c) [a] $\phi(r) = -\int E(r)\,dr = \begin{cases} C_1, & r \le R \\ \frac{Q}{4\pi\varepsilon_0 r} + C_2, & r > R \end{cases}$

$\phi(r \to +\infty) = 0 \Rightarrow C_2 = 0$

$\phi(R^+) = \phi(R^-) \Rightarrow C_1 = \frac{Q}{4\pi\varepsilon_0 R}$

$\phi(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 R}, & r \le R \\ \frac{Q}{4\pi\varepsilon_0 r}, & r > R \end{cases}$

[b] $\phi(r) = -\int E(r)\,dr = \begin{cases} -\frac{Q}{8\pi\varepsilon_0}\frac{r^2}{R^3} + C_1, & r \le R \\ \frac{Q}{4\pi\varepsilon_0 r} + C_2, & r > R \end{cases}$

$\phi(r \to +\infty) = 0 \Rightarrow C_2 = 0$

$\phi(R^+) = \phi(R^-) \Rightarrow \frac{Q}{4\pi\varepsilon_0 R} = -\frac{Q}{8\pi\varepsilon_0 R} + C_1$

$\Rightarrow C_1 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} + \frac{1}{2R}\right) = \frac{3Q}{8\pi\varepsilon_0 R}$

$\phi(r) = \begin{cases} \frac{3Q}{8\pi\varepsilon_0 R} - \frac{Q r^2}{8\pi\varepsilon_0 R^3}, & r \le R \\ \frac{Q}{4\pi\varepsilon_0 r}, & r > R \end{cases} = \begin{cases} \frac{Q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2), & r \le R \\ \frac{Q}{4\pi\varepsilon_0 r}, & r > R \end{cases}$